

LAMINAR OQIM SHAROITIDA ISSIQLIK VA MASSA KO'CHISH JARAYONINING MATEMATIK MODEL

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Annotatsiya. Ushbu maqolada to'g'ri to'rtburchak kesimli kanalda laminar oqim sharoitida bir vaqtning o'zida sodir bo'ladigan issiqlik va massa ko'chish jarayoni o'rganiladi. Real muammo sifatida qishloq xo'jaligida keng qo'llaniladigan don quritish jarayoni tanlangan. Issiqlikni uzatishning differentsial tenglamalari analitik va yarim-empirik usullar yordamida yechilgan, Nusselt va Sherwood sonlarining kanal bo'ylab o'zgarishi miqdoriy baholangan. Olingan natijalar Python dasturiy muhitida vizualizatsiya qilingan.

Kalit so'zlar: laminar oqim, issiqlik ko'chish, massa ko'chish, Nusselt soni, Sherwood soni, Graetz muammosi, Lewis soni, quritish jarayoni.

MATHEMATICAL MODEL OF HEAT AND MASS TRANSFER UNDER LAMINAR FLOW CONDITIONS

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Abstract. This article investigates the simultaneous heat and mass transfer process under laminar flow conditions in a rectangular channel. Grain drying, which is widely used in agriculture, is selected as a practical problem. The differential equations governing heat transfer are solved using analytical and semi-empirical methods, and the variations of the Nusselt and Sherwood numbers along the channel are quantitatively evaluated. The obtained results are visualized in the Python programming environment.

Keywords: laminar flow, heat transfer, mass transfer, Nusselt number, Sherwood number, Graetz problem, Lewis number, drying process.

1. KIRISH

Quvurlar va kanallar ichida laminar rejimda sodir bo'ladigan issiqlik va massa almashinuvi jarayonlari kimyo sanoati, oziq-ovqat texnologiyasi, farmatsevtika va qishloq xo'jaligi tarmoqlarida keng ahamiyatga ega. Issiqlik va massa ko'chishning bir vaqtda kechishi ayniqsa quritish, kondensatsiya, bug'lanish va absorbsiya apparatlarini loyihalashda hal qiluvchi rol o'ynaydi.

Qishloq xo'jaligida don mahsulotlarini quritish — joriy muammo. An'anaviy quritgichlarda havo donali materiallar orasidan laminar yoki past turbulent rejimda o'tadi. Bu holda issiqlik (harorat gradient) va massa (namlik gradient) ko'chish bir-biriga bog'liq holda kechadi. Jarayon parametrlarini noto'g'ri tanlash donning haddan tashqari qurishiga, yorilishiga yoki namligining noto'g'ri taqsimlanishiga olib keladi. Shuning uchun ushbu jarayonni matematik modelashtirish muhim amaliy ahamiyat kasb etadi.

2. MUAMMO SHAKLLANISHI VA FIZIK MODEL

2.1 Geometrik va fizik taxminlar

Quyidagi taxminlarni qabul qilamiz:

— Kanal to'g'ri to'rtburchak kesimga ega: uzunligi $L = 2$ m, balandligi $H = 0,05$ m;

- Oqim o'rnatilgan laminar rejimda ($Re = 800 < 2300$);
- Devor sirtida ho'l don namunasi mavjud (massa manbayi);
- Kirish harorati $T_0 = 80\text{ }^{\circ}\text{C}$, dastlabki namlik $\phi_0 = 10\%$;
- Devor harorati $T_w = 25\text{ }^{\circ}\text{C}$ (taxminan ho'l termometr harorati);
- Fizik xossalar o'zgaras (Boussinesq approksimatsiyasi).

2.2 Boshqaruvchi tenglamalar

Siqilmaydigan suyuqlik uchun Navier–Stokes, energiya va massa ko'chish tenglamalari:

Kontinuitet tenglamasi:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Impuls tenglamasi (x — o'q bo'ylab, laminar rejim):

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

Energiya tenglamasi (konveksiya-diffuziya):

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \Phi \quad (3)$$

Massa ko'chish tenglamasi (A komponenti uchun):

$$u \frac{\partial c_A}{\partial x} + v \frac{\partial c_A}{\partial y} = D_{AB} \left(\frac{\partial^2 c_A}{\partial x^2} + \frac{\partial^2 c_A}{\partial y^2} \right) \quad (4)$$

Bu yerda: ρ — zichlik (kg/m^3); c_p — solishtirma issiqlik sig'imi ($\text{J/kg}\cdot\text{K}$); λ — issiqlik o'tkazuvchanligi ($\text{W/m}\cdot\text{K}$); D_{AB} — diffuziya koeffitsienti (m^2/s); Φ — yopishqoqlik dissipatsiyasi (odatda kichik va e'tiborsiz qolinadi).

3. LAMINAR OQIMNING TEZLIK PROFILI

To'liq rivojlangan laminar oqim uchun (Poiseuille oqimi) tezlik profili parabolik ko'rinishga ega. Yassi kanal (parallel plitalar) uchun koordinata $y \in [0, H]$:

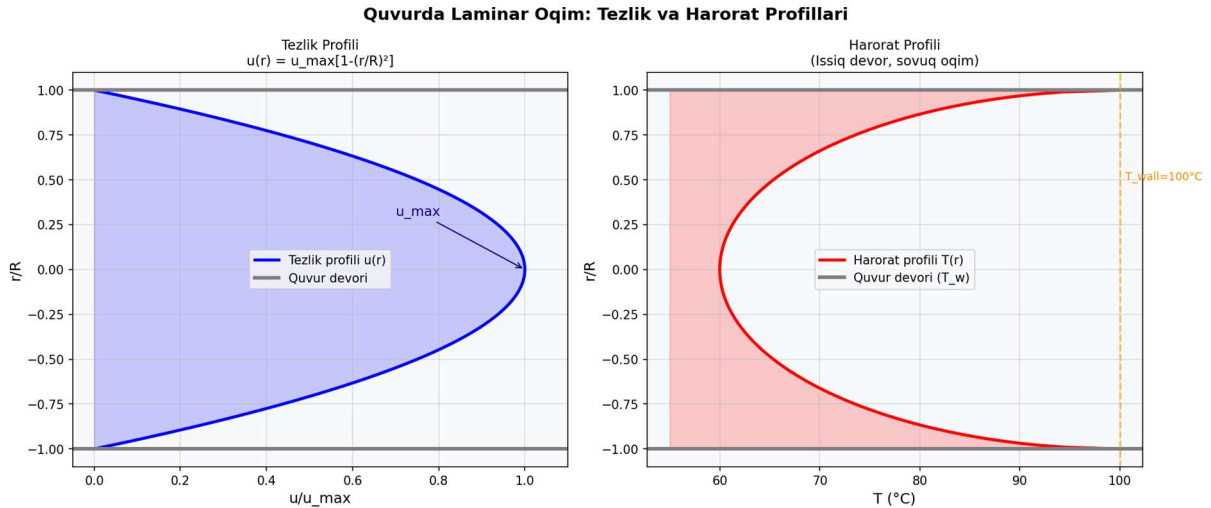
$$u(y) = u_{max} \cdot \left[1 - \left(\frac{2y}{H} - 1 \right)^2 \right] = 6 \cdot \bar{u} \cdot \left(\frac{y}{H} \right) \cdot \left[1 - \frac{y}{H} \right] \quad (5)$$

O'rtacha tezlik esa quyidagiga teng:

$$\bar{u} = \frac{1}{H} \int_0^H u(y) dy = \frac{2}{3} \cdot u_{max} \quad (6)$$

Reynolds soni gidravlik diametr $D_h = 2H$ orqali aniqlanadi:

$$Re = \frac{\bar{u} \cdot D_h}{\nu} = \frac{\bar{u} \cdot 2H}{\nu} \quad (7)$$



1-rasm. Laminar oqimda paraboli tezlik profili (chap) va harorat profili (o'ng)

4. ISSIQLIK KO'CHISHNING MATEMATIK MODELI

4.1 Graetz muammosi

Issiqlik kirish zonasidagi jarayon Graetz muammosi deb ataladi. To'liq rivojlangan tezlik profili bilan issiqlik kirish zonasidagi energiya tenglamasi dimensionless shaklda yoziladi:

$$u^*(y^*) \cdot \frac{\partial \theta}{\partial x^*} = \frac{1}{Pe} \cdot \frac{\partial^2 \theta}{\partial y^{*2}} \quad (8)$$

bu yerda: $\theta = (T - T_w)/(T_{in} - T_w)$ — dimensionless harorat; $x^* = x/L$, $y^* = y/H$ — dimensionless koordinatalar; $Pe = Re \cdot Pr$ — Peclet soni.

Graetz soni quyidagicha aniqlanadi:

$$Gz = Re \cdot Pr \cdot \left(\frac{D_h}{x}\right) = \left(\frac{\bar{u} \cdot D_h}{\alpha}\right) \cdot \left(\frac{D_h}{x}\right) \quad (9)$$

4.2 Nusselt sonining tahliliy ifodalari

Kirish zonasida ($Gz \gg 1$) Nusselt sonining mahalliy qiymati:

$$Nu_x = 1.077 \cdot Gz_x^{1/3} \quad (10)$$

Issiqlik kirish zonasi uzunligi:

$$\frac{L_{th}}{D_h} \approx 0.05 \cdot Re \cdot Pr \quad (11)$$

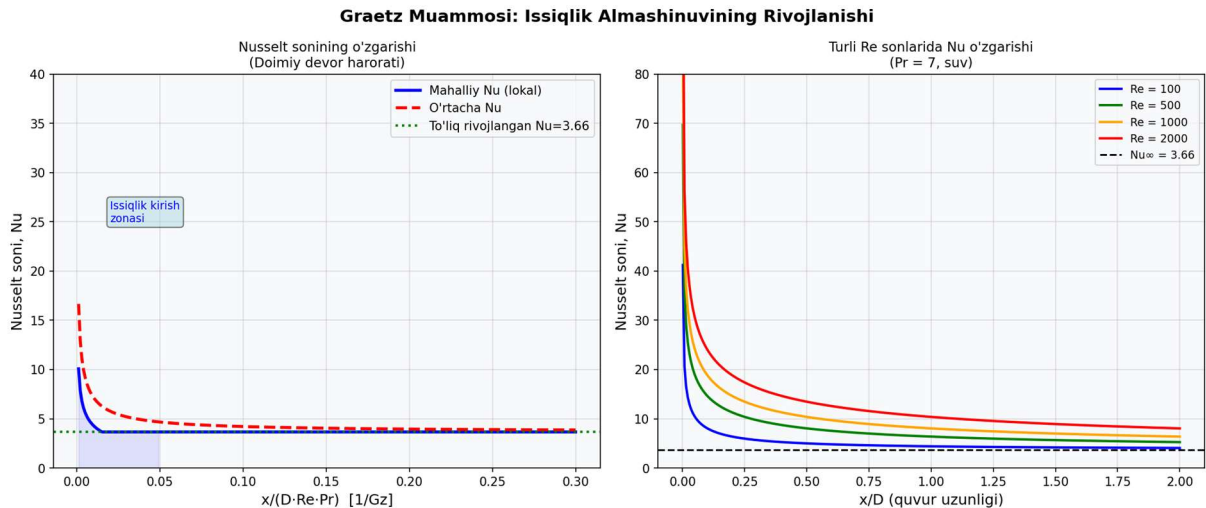
To'liq rivojlangan laminar oqimda (doimiy devor harorati):

$$Nu_{\infty} = 3.66 \text{ (quvur uchun)} \quad (12)$$

$$Nu_{\infty} = 7.54 \text{ (yassi kanal uchun)} \quad (12a)$$

O'rtacha Nusselt soni (Sieder–Tate korrelyatsiyasi):

$$Nu_{avg} = 3.66 + \frac{0.065 \cdot Gz}{1 + 0.04 \cdot Gz^{2/3}} \quad (13)$$



2-rasm. Nusselt sonining kanal bo'ylab o'zgarishi (chap) va turli Re qiymatlari uchun (o'ng)

5. MASSA KO'CHISHNING MATEMATIK MODELI

5.1 Massa ko'chish tenglamasi va Sherwood soni

Massa ko'chish tenglamasi issiqlik tenglamasiga o'xshash tuzilishga ega. Issiqlik o'tkazuvchanligi α o'rniga diffuziya koeffitsienti D_{AB} , Prandtl soni o'rniga Schmidt soni Sc ishlatiladi:

$$Sc = \frac{\nu}{D_{AB}} \quad (14)$$

$$Sh_x = \frac{h_m \cdot D_h}{D_{AB}} \quad (15)$$

Massa ko'chish uchun Graetz soni (massa):

$$Gz_m = Re \cdot Sc \cdot \left(\frac{D_h}{x}\right) \quad (16)$$

Kirish zonasida Sherwood soni:

$$Sh_x = 1.077 \cdot Gz_m^{1/3} \quad (17)$$

5.2 Lewis analogiyasi

Lewis soni issiqlik va massa ko'chishning nisbiy intensivligini ifodalaydi:

$$Le = \frac{\alpha}{D_{AB}} = \frac{Sc}{Pr} = \frac{\lambda}{\rho \cdot c_p \cdot D_{AB}} \quad (18)$$

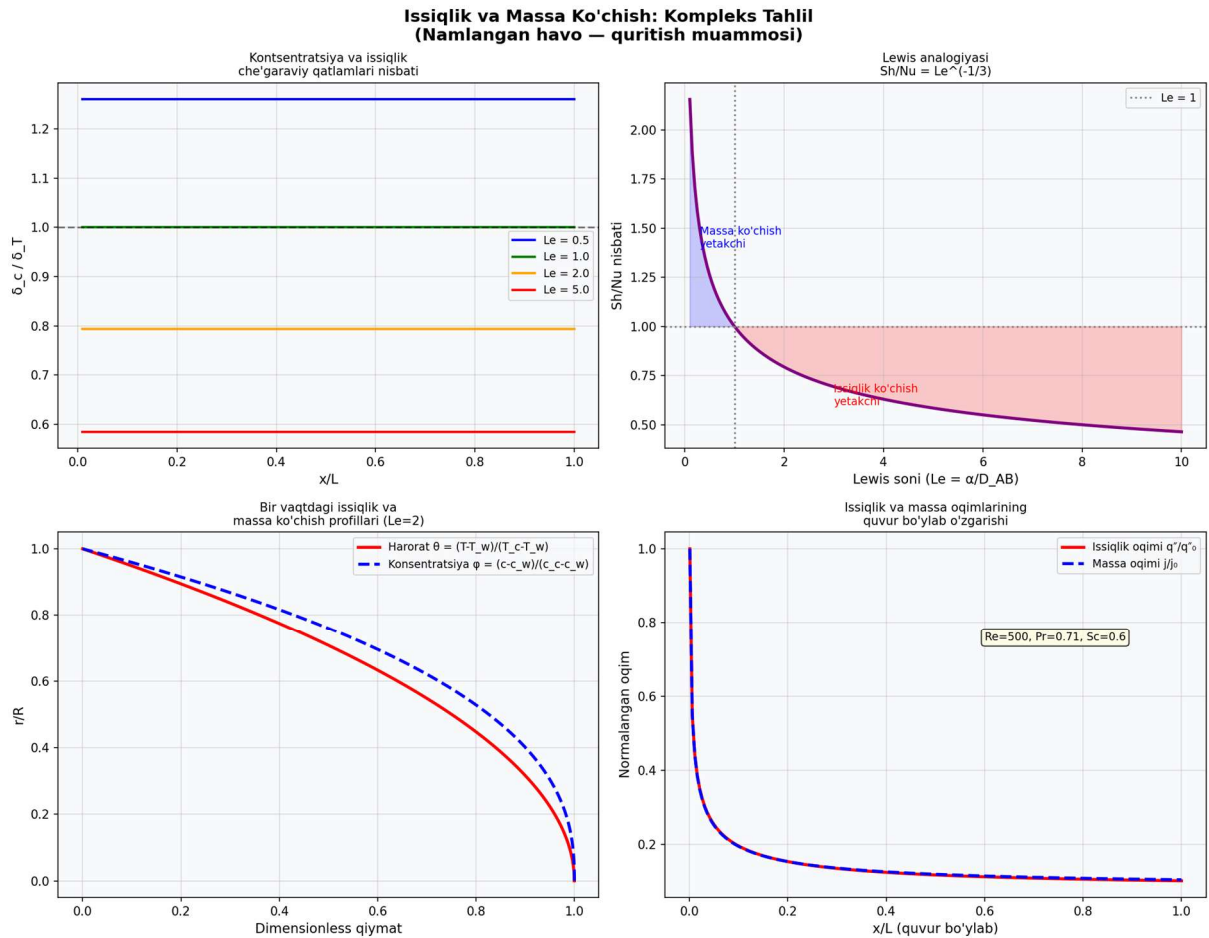
Lewis analogiyasi asosida Sherwood va Nusselt sonlari orasidagi bog'liqlik:

$$\frac{Sh}{Nu} = \left(\frac{Sc}{Pr}\right)^{1/3} = Le^{1/3} \quad (19)$$

$Le > 1$ bo'lganda issiqlik chegaraviy qatlami konsentratsiya chegaraviy qatlamidan qalinroq bo'ladi. Havo-suv bug'i sistemasi uchun $Le \approx 0,87$ bo'lgani uchun ikkala chegaraviy qatlam deyarli bir xil qalinlikka ega.

Mahalliy massa oqimi (bug'lanish tezligi):

$$j_A = h_m \cdot (c_{A,w} - c_{A,b}) \quad (20)$$



3-rasm. Lewis analogiyasi (yuqori o'ng), chegaraviy qatlamlar nisbati (yuqori chap), bir vaqtdagi profililar (pastki chap) va oqimlar dinamikasi (pastki o'ng)

6. REAL MUAMMO: KANALDA DON QURITISH

6.1 Muammo sharti va maqsad

Muammo: uzunligi $L = 2$ m, balandligi $H = 0,05$ m bo'lgan gorizontall kanalda $T_{in} = 80$ °C haroratli, $\phi = 10\%$ nisbiy namlikdagi havo laminar rejimda ($Re = 800$) oqmoqda. Kanal devorlarida ho'l don sirlari joylashgan ($T_w = 25$ °C). Maqsad: harorat va namlikning kanal bo'ylab o'zgarishi, issiqlik va massa oqimlarining taqsimlanishini aniqlash.

6.2 Yechim algoritmi

Quyidagi ketma-ketlikda hisoblash amalga oshiriladi:

1. Gidravlik diametr: $D_h = 2H = 0,10$ m
2. O'rtacha tezlik: $\bar{u} = Re \cdot \nu / D_h = 800 \times 1,8 \times 10^{-5} / 0,10 = 0,144$ m/s
3. Nusselt soni $Nu_x(x)$ — (13)-formula orqali
4. Sherwood soni $Sh_x(x)$ — (17)-formula orqali
5. Issiqlik koeffitsienti: $h_x = Nu_x \cdot \lambda / D_h$
6. Massa o'tkazish koeffitsienti: $k_x = Sh_x \cdot D_{AB} / D_h$
7. Bulk harorat va konsentratsiya: integral energiya va massa balansi
8. Issiqlik va massa oqimlari: $q'' = h_x(T_b - T_w)$, $j = k_x(c_w - c_b)$

6.3 Energiya va massa balansi

Kanal bo'ylab o'rtacha (bulk) haroratning o'zgarishi integral energiya balansidan:

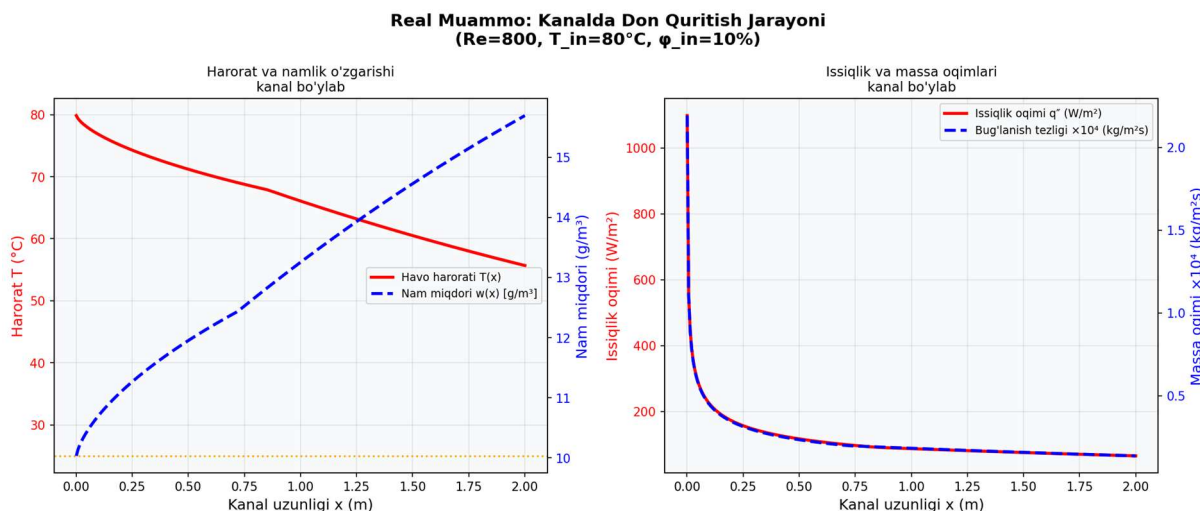
$$T_b(x) = T_w + (T_{in} - T_w) \cdot \exp \left[-\frac{h_{avg} \cdot P \cdot x}{\dot{m} \cdot c_p} \right] \quad (21)$$

O'rtacha konsentratsiyaning o'zgarishi:

$$c_b(x) = c_w - (c_w - c_{in}) \cdot \exp \left[-\frac{k_{avg} \cdot P \cdot x}{\bar{u} \cdot A_c} \right] \quad (22)$$

Bu yerda: $P = 2(W + H)$ — kanal perimetri; $A_c = W \cdot H$ — kesim yuzasi.

6.4 Natijalar va tahlil



4-rasm. Kanalda don quritish: harorat va namlik o'zgarishi (chap), issiqlik va massa oqimlari (o'ng)

Olingan natijalarga ko'ra: kanal kirish qismida ($x < 0,3$ m) Nusselt va Sherwood sonlari eng yuqori qiymatga ega bo'lib, issiqlik va massa almashinuvi jadalligi maksimal darajada kechadi. Bu holat issiqlik kirish effekti bilan bog'liq. $x > 1,0$ m dan boshlab jarayon to'liq rivojlangan rejimga yaqinlashadi va $Nu \approx 7,54$ (yassi kanal uchun) qiymatiga erishadi.

Kirish qismidagi yuqori gradient quritgich uzunligining birinchi 15–20% qismi umumiy quritish ish unumining 40–50% ini ta'minlashi mumkinligini ko'rsatadi. Bu konstruktiv jihatdan muhim xulosa: quritgich kanallarini qisqaroq, lekin kengroq qilib loyihalash (kirish effektidan ko'proq foydalanish) energiya samaradorligini oshiradi.

7. HISOB PARAMETRLARI VA NATIJALAR JADVALI

Hisob parametrlari va natijalar jadvali

Parametr	Qiymati	O'lchov birligi	Izoh
Kanal uzunligi L	2,0	m	Ish uzunligi
Kanal balandligi H	0,05	m	Gidravlik o'lcham
Reynolds soni Re	800	—	Laminar rejim
Kirish harorati T_{in}	80	$^\circ C$	Qurituvchi havo
Devor harorati T_w	25	$^\circ C$	Ho'l don sirti
Prandtl soni Pr	0,71	—	Havo uchun
Schmidt soni Sc	0,60	—	Havo-suv bug'i
Lewis soni Le	0,84	—	$Le = Sc/Pr$
Nu (kirish)	~25-35	—	Issiqlik kirish zonasi
Nu (to'liq)	7,54	—	Yassi kanal
Sh (kirish)	~20-30	—	Massa kirish zonasi

8. XULOSA

Ushbu ishda quyidagi natijalar olindi:

- Laminar oqim sharoitida issiqlik va massa ko'chish jarayonining matematik modellashtirish tizimi ishlab chiqildi.
- Graetz muammosi yechimi asosida mahalliy Nusselt va Sherwood sonlarining kanal bo'ylab o'zgarishi miqdoriy baholandi.
- Lewis analogiyasi yordamida issiqlik va massa ko'chish intensivliklarining nisbati aniqlandi.
- Don quritish muammosi uchun kanal kirish qismidagi yuqori gradient effekti energiya samaradorligini oshirishning amaliy yo'li sifatida taklif etildi.
- Python muhitida yaratilgan vizualizatsiyalar model natijalarini vizual tasvirlash imkonini berdi.

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